Hall Ticket No: | | | | | | | | | | | **Question Paper Code :**

ANIL NEERUKONDA INSTITUTE OF TECHNOLOGY & SCIENCES
(AUTONOMOUS)

B. Tech I Semester Regular Examinations November - 2015

(Regulations: R15)

Engineering Mathematics-I

Date: **Time: 3 hours** **Max Marks: 60**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. a) i) If $z = f(x+ct) + \phi(x-ct)$, *prove that* $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$. (2)

ii) *If* $x = r\cos\theta, y = r\sin\theta$, *show that* $(i) \frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$ $(ii) \frac{1}{r}\frac{\partial x}{\partial \theta} = r\frac{\partial \theta}{\partial x}$ (2)

b) *If* $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, *evaluate* $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. (8)

OR

2. a) *If* $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, *show that* $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$. (4)

b) Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 4$. (8)

UNIT – II

3. a) (i)Write the conditions for a Fourier Expansion (2)

(ii) Find a_n in the Fourier expansion of the function $f(x) = x^2$, $-\pi < x < \pi$ (2)

b) *Expand* $f(x) = \sqrt{(1-\cos x)}, 0 < x < 2\pi$ *in a Fourier series.*
Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$ (8)

OR

4. a) Find Half range cosine series of $f(x) = x$ *in* $0 < x < 2.$ (4)

b) By using the sine series for f(x) = 1 in $0 < x < \pi$, *show that* $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ (8)

UNIT – III

5. a) Find the equation of the sphere through the points (1 , 0 , 0) , (0 , 1, 0) , (0, 0, 1) And has its centre on the plane x + y + z = 6 . (4)

b) Find the equation of the sphere which touches the plane x - 2y – 2z = 7 at the point (3, -1, -1) and passes through the point (1, 1, -3) (8)

OR

6. a) Show that , the spheres $x^2+y^2+z^2+6y+14z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ Intersect at right angels. Find their plane of intersection. (4)

b) Find the equation of the enveloping cone of the sphere $x^2+y^2+z^2=a^2$ with the vertex at the point (x_1,y_1,z_1). (8)

UNIT – IV

7. a) i) Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dxdy$ (2)

ii) Evaluate $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r\sin\theta\,drd\theta$ (2)

b) Change the order of integration in $I=\int_0^1\int_x^{\sqrt{(2-x^2)}}\frac{x}{\sqrt{(x^2+y^2)}}\,dxdy$ and hence evaluate. (8)

OR

8. a) Evaluate $\int_0^a\int_0^x\int_0^{x+y}e^{x+y+z}\,dzdydx$. (4)

b) Evaluate $\iint_D xy\sqrt{(1-x-y)}\,dxdy$ where D is the region bounded by x = 0, y = 0, and x + y = 1using the transformation x + y = u, y = uv. (8)

UNIT – V

9. a) Evaluate $\int_0^1\frac{x^2}{\sqrt{(1-x^4)}}\,dx$. . (4)

b) State & Prove, the relation between beta and gamma function . (8)

OR

10. a) Define error function. Find the value of (i) erf(x) + erf(-x)

(ii) erfc(x) + erfc(-x) (4)

b) Show that , $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1}dy=\frac{\Gamma(p)}{q^p}$, *where* $p>0, q>0$. (8)

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Date: **Time: 3 hours** **Max Marks: 60**

Answer ONE Question from each Unit
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UNIT – I

1. a) i) If $u = lx + my, v = mx - ly$ then find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v$. (2)

ii) Writhe the conditions for f(x,y) to have minimum and maximum values. (2)

b) i) If $u = f(r,s,t)$ and $r = x/y, s = y/z, r = z/x$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. (4)

ii) Find the dimensions of rectangular box , open at the top, of maximum capacity whose surface is 432 sq.cm. (4)

OR

2. a) i) If $u = x\log xy$ where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$. (2)

ii) In polar coordinates, $x = r\cos\theta, y = r\sin\theta$, show that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$. (2).

b) i) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u.\cos 2u}{4\cos^3 u}$$. (8)

UNIT – II

3. a) Prove that

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2},\ -\pi < x < \pi \ \ and\ hence\ show that \sum\frac{1}{n^2} = \frac{\pi^2}{6}$$ (4)

b) Given $f(x) = -x+1 \quad for -\pi \le x \le 0$ (or) $f(x) = |x| + 1 \quad -\pi < x < \pi$ Is the

$= x+1 \quad for\ \ 0 \le x \le \pi$

function even or odd? Find the Fourier series for $f(x)$ and find the value of

$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+......$ (8)

OR

4. a) Find Half range sine series of $f(x) = x \; in \quad 0 < x < 2.$ (4)

b) Prove that in $0 < x < l, \quad x = \frac{l}{2} - \frac{4l}{\pi^2}(\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + ...)$ and

deduce that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+..... = \frac{\pi^4}{90}$ (8)

UNIT – III

5. a) Find the equation of the sphere through the points (0 , 0 , 0) , (0 , 1, -1) , (-1, 2, 0) (1 ,2 , 3) and hence locate its centre and radius . (4)

b) Find the centre and radius of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11 , x + 2y + 2z = 15$. (8)

OR

6. a) Show that , the spheres $x^2 + y^2 + z^2 + 6y + 14z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ Intersect at right angles. Find their plane of intersection. (4)

b) The radius of a normal section of a right circular cylinder is 2 units , the axis lies along the straight line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$. Find its equation. (8)

UNIT – IV

7. a) i) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\,dx\,dy$ (2)

ii) Evaluate $\int_0^{\pi} \int_0^{a \sin\theta} r\,dr\,d\theta$ (2)

b) Change the order of integration in I = $\int_0^1 \int_{x^2}^{2-x} xy\;dx\,dy$ and hence evaluate. (8)

OR

8. a) Evaluate $\iint r \sin\theta\,dr\,d\theta$ over the cardioid $r = a(1-\cos\theta)$ above the initial line . (4)

b) Evaluate $\iint_R (x+y)^2 \, dx\, dy$ where R is the parallelogram in the xy-plane with vertices (1, 0) , (3, 1), (2,2), (0,1) using the transformation u = x + y and v = x - 2y (8)

UNIT – V

9. a) Evaluate $\int_0^{\infty} e^{-x^2} \, dx$. (4)

b) State & Prove , the relation between beta and gamma function . (8)

OR

10. a) Evaluate $\int_0^{\infty} \sqrt{x} \; e^{-x^3} \, dx$. (4)

b) Show that , $\int_a^b (x-a)^{m-1} (b-x)^{n-1} \, dx = (b-a)^{m+n-1} \beta(m,n)$ (8)