

**Advanced Control Systems**  
(Control Systems Engineering)

Date:

Time: 3 hours

Max Marks: 60

Answer ONE Question from each unit

All questions carry equal marks

All parts of the question must be answered at one place only

**UNIT-I**

1.a) Explain why state variable approach is known as modern control approach. 6M

b) Define and explain state, state variable and Non uniqueness of state models 6M

(OR)

2.a) Prove that a system described by state model of the form 6M

$$\dot{x}(t)=Ax(t)+B u(t)$$

$$y(t)=Cx(t)+Du(t); \text{where } A,B,C,D \text{ constants matrices and time invariants.}$$

b) A System is described by the equation 6M

$$x(k+1)=\begin{bmatrix} -3 & 1 & 0 \\ -4 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}x(k)+\begin{bmatrix} -3 \\ -7 \\ 0 \end{bmatrix}u(k)$$

$x(0)=x^0$ , Using Z-transform technique, transform the state equation into a set of linear algebraic equations in the form

$$\hat{x}(z)=\hat{G}(z)x^0+\hat{H}(z)\hat{u}(z)$$

**UNIT-II**

3.a) Find the state transition matrix of the following homogeneous equation. 6M

$$\dot{x}(t)=\begin{bmatrix} t & 1 \\ 1 & t \end{bmatrix}x(t)$$

b) Consider the state equation 6M

$$\dot{x}(t)=\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}x(t). \text{ Find a set of states } x_1(1), x_2(1) \text{ such that } x_1(2)=2$$

(OR)

4.a) Derive an expression to find the solution to state equation using the solution to differential equation. 6M

b) For the state equation.  $\dot{x}(t)=\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}x(t)$ . Find the initial condition vector 6M

$x(0)$ =which will only excite the mode corresponding to the eigen value with the most negative real part.

**UNIT-III**

5.a) Explain controllable canonical form of the state space representation. 6M

b) Investigate the controllability and observability of the following system. 6M

$$\dot{x}(t)=\begin{bmatrix} 1 & 0 \\ 0 & 2t \end{bmatrix}x(t)+\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$Y(t)=[0 \ 1] x(t)$$

(OR)

6.a) Consider the system

$$X(k+1) = \begin{bmatrix} 1 & k \\ 0 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ k \end{bmatrix} u(k) \quad 6M$$

In this system controllable at  $k=0$ ? If Yes, find a control sequence to drive the system from

$$x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ to } x^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) Consider the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} u(t) \quad 6M$$

Find, if possible a control law to transfer the state from

$$x(1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ to } x(4) = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

#### UNIT-IV

7.a) Define and explain the significance of equilibrium points. 6M

b) Consider the linear autonomous system 6M

$$x(k+1) = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix} x(k).$$

Using direct method of Lyapunov, determine stability of the equilibrium state.

(OR)

8.a) Explain the Lyapunov's instability theorem 6M

b) Construct Lyapunov's function for the following 6M

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1^2 x_2 - 2x_1^3 \text{ and determine its stability.}$$

#### UNIT-V

9.a) Show that the zeros of a scalar system are invariant under linear state feedback to the input. 6M

b) Consider the state model 6M

$$\dot{X} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u \quad Y = [40/3 \quad -15 \quad 5/3]$$

Determine the transfer function  $\hat{Y}(s)/\hat{U}(s)$ .

10.a) Explain the design of prediction observer. 6M

b) Explain the necessary and sufficient condition for arbitrary pole- placement. 6M

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